STEP II, 2019, Q8 EC

Many good solutions were seen to this question, but solutions often lacked clear enough justification to be awarded full marks. However, there were also a surprising number of candidates who did not manage to invert the 2x2 matrices successfully. Candidates who claimed that the function f was the determinant of the matrix were not able to score high marks as the solutions did not then demonstrate that the results were true of *any* function satisfying the property given.

The first two parts of this question were largely done well. The third part was found more difficult, with few candidates realising that $\begin{pmatrix} a & b \\ ka & kb \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} a & b \\ a & b \end{pmatrix}$. Those who did were then often able to provide a full solution, although often these were not fully justified. Several candidates instead used $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} a & b \\ k^{-1}a & k^{-1}b \end{pmatrix} = \begin{pmatrix} a & b \\ a & b \end{pmatrix}$ to produce a solution which covered all cases apart from the one where $k = 0$. In some cases, candidates did not appear to consider $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ to be an example of a matrix in which the second row was a multiple of the first.

In part (iv) many candidates made use of the fact that $f(P) \neq 0$ without showing that this must be the case.