

STEP II, 2019, Q7

- 7 (i) The points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} , respectively. Each of these vectors is a unit vector (so $\mathbf{a} \cdot \mathbf{a} = 1$, for example) and

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}.$$

Show that $\mathbf{a} \cdot \mathbf{b} = -\frac{1}{2}$. What can be said about the triangle ABC ? You should justify your answer.

- (ii) The four distinct points A_i ($i = 1, 2, 3, 4$) have unit position vectors \mathbf{a}_i and

$$\sum_{i=1}^4 \mathbf{a}_i = \mathbf{0}.$$

Show that $\mathbf{a}_1 \cdot \mathbf{a}_2 = \mathbf{a}_3 \cdot \mathbf{a}_4$.

- (a) Given that the four points lie in a plane, determine the shape of the quadrilateral with vertices A_1 , A_2 , A_3 and A_4 .
- (b) Given instead that the four points are the vertices of a regular tetrahedron, find the length of the sides of this tetrahedron.



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