STEP II, 2019, Q6 MS

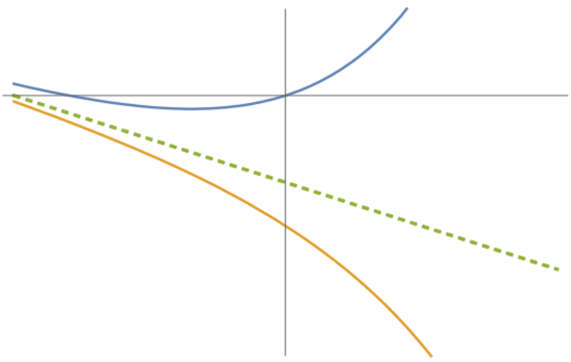
In the first part, substituting $y = mx + c$ into the differential equation will allow the values of m and c to be deduced. Since stationary points must satisfy $\frac{dy}{dx} = 0$, substituting this into the differential equation shows that stationary points must lie on the given line. It then follows that solution curves with $k < 2$ cannot have stationary points as they would have to cross the straight-line solution that has already been found.

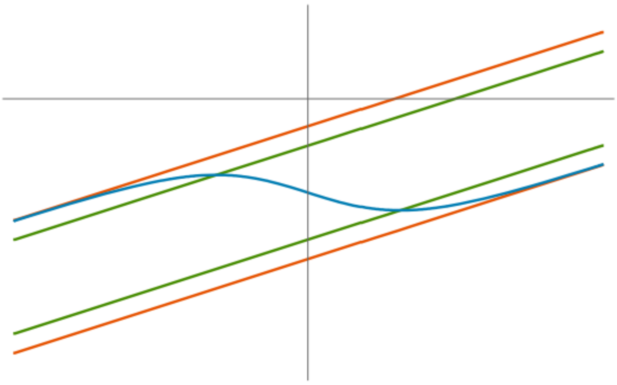
Given that the relationship between x and y for any stationary point is known, it is possible to differentiate the differential equation and evaluate $\frac{d^2y}{dx^2}$ for any stationary point.

Once the substitution provided has been applied, the new differential equation can be solved by separating the variables and have equations that can be sketched easily.

In the second part, the same approach as part (i) can be used to find the possible sets of values for m and c. The RHS of the differential equation can be considered a function of $y - x$ and this allows it to be factorised. Solving $\frac{dy}{dx} = 0$ then shows that x and y must satisfy one of two linear equations and the sign of $\frac{dy}{dx}$ can be deduced for points between these two lines.

The graph can then be sketched, remembering that the curve cannot cross the two straight-line solutions.

6	(i)		If $y = mx + c$, Then the differential equation becomes	
			$m = mx + c + x + 1$	M1
			$m = -1, c = -2$ $y = -x - 2$	A1
			$\frac{dy}{dx} = 0 \Rightarrow y + x + 1 = 0 \Rightarrow y = -x - 1$	E1 (AG)
			$y = y_3(x)$ cannot cross the line $y = -x - 2$. So if $y_3(0) < -2$, it cannot reach the line $y = -x - 1$ and hence has no stationary points.	E1
			At a stationary point, $\frac{d^2y}{dx^2} = \frac{dy}{dx} + 1 = y + x + 2 = 1 > 0$ so minimum	E1
			$\frac{dY}{dx} = Y + 2$	M1
			$\log(Y + 2) = x + c$	M1
			$Y = -2 + Ae^x$ $y = -x - 2 + Ae^x$	A1
			$y(0) = 0 \Rightarrow A = 2$ $y(0) = -3 \Rightarrow A = -1$ (attempt at both)	M1
			So $y = -x - 2 + 2e^x$ So $y = -x - 2 - e^x$ (both)	
				
			Curves tending to asymptote to the left Curve above line through origin tending to ∞ Curve below line tending to $-\infty$	G1 G1 G1
				(12 marks)
	(ii)		If $y = mx + c$, Then the differential equation becomes $m = (mx + c)^2 + 4(mx + c) + x^2 - 4x - 2x(mx + c) + 3$ $0 = (m^2 - 2x + 1)x^2 + (2mc + 4m - 4 - 2c) + c^2 + 4c + 3 - m$	
			From x^2: $m = 1$ From x: $2mc + 4m - 4 - 2c = 2c + 4 - 4 - 2c = 0$	

			From 1: $c^2 + 4c + 2 = 0 \Rightarrow c = -2 \pm \sqrt{2}$ Any of these equations Correct values of m and c Solutions: $y = x - 2 \pm \sqrt{2}$ $\frac{dy}{dx} = (y - x)^2 + 4(y - x) + 3$ (writing as a function of $y - x$) $= (y - x + 3)(y - x + 1)$ Stationary pts: $y = x - 1$ or $y = x - 3$ Between these lines the gradient is negative. (Correctly justified) So stationary points on $y = x - 1$ are maxima and stationary on $y = x - 3$ are minima.  Curve does not intersect other solutions Curve has stationary points on correct lines	 M1 A1 M1 A1 A1 A1 G1 G1 (8 marks)

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