# STEP II, 2019, Q6

6 **Note**: You may assume that if the functions $y_1(x)$ and $y_2(x)$ both satisfy one of the differential equations in this question, then the curves $y = y_1(x)$ and $y = y_2(x)$ do not intersect.

**(i)** Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y + x + 1$$

that has the form $y = mx + c$, where $m$ and $c$ are constants.

Let $y_3(x)$ be the solution of this differential equation with $y_3(0) = k$. Show that any stationary point on the curve $y = y_3(x)$ lies on the line $y = -x - 1$. Deduce that solution curves with $k < -2$ cannot have any stationary points.

Show further that any stationary point on the solution curve is a local minimum.

Use the substitution $Y = y + x$ to solve the differential equation, and sketch, on the same axes, the solutions with $k = 0$, $k = -2$ and $k = -3$.

**(ii)** Find the two solutions of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^2 - 2xy - 4x + 4y + 3$$

that have the form $y = mx + c$.

Let $y_4(x)$ be the solution of this differential equation with $y_4(0) = -2$. (Do not attempt to find this solution.)

Show that any stationary point on the curve $y = y_4(x)$ lies on one of two lines that you should identify. What can be said about the gradient of the curve at points between these lines?

Sketch the curve $y = y_4(x)$. You should include on your sketch the two straight line solutions and the two lines of stationary points.