STEP II, 2019, Q5 EC

It was difficult to get full marks on this question, with most candidates struggling to correctly prove 'if and only if' statements in both directions.

Mostly, the two constant sequences were successfully found and then correctly rejected for sequences of period 2, but few thought to check that the other two solutions to the quartic did not also coincide with the constant sequences. Most candidates were able to use the discriminant to produce bounds on p, but many could not justify the strictness of the inequality, which was best done by considering the boundary cases separately.

The first request of the second part was answered well, with most using only the fact that it was a positive quadratic and a minority delving into the details of $f(x)$. Most candidates who reached this part of the questions correctly used the result $f(x) > x$ to show that $f\big(f(x)\big) = x$ has no solutions, but many overlooked the connection between the final part and part (i).