

STEP II, 2019, Q5

- 5 The sequence u_0, u_1, \dots is said to be a *constant sequence* if $u_n = u_{n+1}$ for $n = 0, 1, 2, \dots$. The sequence is said to be a *sequence of period 2* if $u_n = u_{n+2}$ for $n = 0, 1, 2, \dots$ and the sequence is not constant.

- (i) A sequence of real numbers is defined by $u_0 = a$ and $u_{n+1} = f(u_n)$ for $n = 0, 1, 2, \dots$, where

$$f(x) = p + (x - p)x,$$

and p is a given real number.

Find the values of a for which the sequence is constant.

Show that the sequence has period 2 for some value of a if and only if $p > 3$ or $p < -1$.

- (ii) A sequence of real numbers is defined by $u_0 = a$ and $u_{n+1} = f(u_n)$ for $n = 0, 1, 2, \dots$, where

$$f(x) = q + (x - p)x,$$

and p and q are given real numbers.

Show that there is no value of a for which the sequence is constant if and only if $f(x) > x$ for all x .

Deduce that, if there is no value of a for which the sequence is constant, then there is no value of a for which the sequence has period 2.

Is it true that, if there is no value of a for which the sequence has period 2, then there is no value of a for which the sequence is constant?



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