

STEP II, 2019, Q3

3 For any two real numbers x_1 and x_2 , show that

$$|x_1 + x_2| \leq |x_1| + |x_2|.$$

Show further that, for any real numbers x_1, x_2, \dots, x_n ,

$$|x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|.$$

(i) The polynomial f is defined by

$$f(x) = 1 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + x^n$$

where the coefficients are real and satisfy $|a_i| \leq A$ for $i = 1, 2, \dots, n-1$, where $A \geq 1$.

(a) If $|x| < 1$, show that

$$|f(x) - 1| \leq \frac{A|x|}{1 - |x|}.$$

(b) Let ω be a real root of f , so that $f(\omega) = 0$. In the case $|\omega| < 1$, show that

$$\frac{1}{1 + A} \leq |\omega| \leq 1 + A. \quad (*)$$

(c) Show further that the inequalities (*) also hold if $|\omega| \geq 1$.

(ii) Find the integer root or roots of the quintic equation

$$135x^5 - 135x^4 - 100x^3 - 91x^2 - 126x + 135 = 0.$$



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