STEP II, 2019, Q2 MS

From a sketch of the function, it can be seen that the first integral corresponds to an area below the curve and the second integral corresponds to an area to the left of the curve. These two areas make a rectangle, whose area is clearly expressed by the expression on the right of the equation.

In the first part it is clear that the value of $g(0)$ satisfies $\left(g(0)\right)^3 + g(0) = 0$. Clearly $g(0) = 0$ satisfies this, but it is necessary to factorise and then show that the quadratic factor has no other real solutions. The second result can be seen by differentiating and observing that $(3g(t)^2 + 1)$ must be greater than 0.

To evaluate the integral, observe that $g^{-1}(s) = s^3 + s$ and apply the result shown at the start of the question.

For the second part it must be noted that this function does not satisfy the conditions for the initial result to be applied. However, it can be seen that $h(t) = g(t + 2)$.

It therefore follows that $h'(t) > 0$ and the values of $h(0)$ and $h(8)$ can be deduced. By considering a sketch of this function it can be seen how to modify the initial result to apply in this case.

2			Sketch with areas $\int_0^x f(t)\,dt$, $\int_0^{f(x)} f^{-1}(y)\,dy$ and rectangle correctly identified. (One mark any one)	G1 G1 (2 marks)
	(i)		$g(0)(g(0)^2+1)=0$ factorised $g(0)$ real so $g(0)=0$ (must be justified) $1=(3g(t)^2+1)g'(t)$ $(3g(t)^2+1)>0$ so $g'(t)>0$ $g(2)^3+g(2)-2=0$ $(g(2)-1)(g(2)^2+g(2)+2)=0$ $\Delta=-7<0$ so $g(2)=1$ or g(2)>0 justified $g^{-1}(s)=s^3+s$ $\int_0^2 g(t)dt=2g(2)-\int_0^{g(2)} g^{-1}(s)ds$ $=\frac{5}{4}$	M1 A1 (AG) M1 A1 (AG) M1 A1 B1 M1 A1 (9 marks)
	(ii)		$h(t)=g(t+2)$ so $h(0)=g(2)=1$ and $h'(t)>0$ $(h(8)-2)(h(8)^2+2h(8)+5)=0$ $h(8)=2$ correctly justified $h^{-1}(s)=s^3+s-2$ $\int_0^8 h(t)dt+\int_{h(0)}^{h(8)} h^{-1}(s)ds=16$ (or similar correct equation) $\int_0^8 h(t)dt=16-\int_1^2(s^3+s-2)ds$ $=16-[\frac{s^4}{4}+\frac{s^2}{2}-2s]_1^2$ (integration) $=12\frac{3}{4}$	M1 A1 M1 A1 B1 M1 A1 M1 A1 (9 marks)