

## STEP II, 2019, Q2

- 2 The function  $f$  satisfies  $f(0) = 0$  and  $f'(t) > 0$  for  $t > 0$ . Show by means of a sketch that, for  $x > 0$ ,

$$\int_0^x f(t) dt + \int_0^{f(x)} f^{-1}(y) dy = xf(x).$$

- (i) The (real) function  $g$  is defined, for all  $t$ , by

$$(g(t))^3 + g(t) = t.$$

Prove that  $g(0) = 0$ , and that  $g'(t) > 0$  for all  $t$ .

Evaluate  $\int_0^2 g(t) dt$ .

- (ii) The (real) function  $h$  is defined, for all  $t$ , by

$$(h(t))^3 + h(t) = t + 2.$$

Evaluate  $\int_0^8 h(t) dt$ .



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