STEP II, 2019, Q1 MS

For the introductory part, first find an equation of the tangent to the curve at the point with $x = a$. An expression can then be found for the y-coordinate of the point on the tangent where $x = p$ and this can easily be shown to be equal to 0 if and only if $g'(a) = 0$.

In part (i), the first result follows by identifying that $g(x) = A(x - q)(x - r)$ allows the first result to be applied. The gradient of the tangent can be found by differentiating $f(x)$ and then the fact that $2a = q + r$ can be used to eliminate a from this expression.

In part (ii) the tangent at the point where $x = c$ is essentially another case of the tangent considered in part (i), so the gradient of this tangent can be deduced easily. By equating the gradients of the two tangents it can be deduced that $q - p = r - q$ (although care needs to be taken to justify the choice of square roots). The equation of the tangent at $x = q$ can also be found and so any other points of intersection between this tangent and the curve can be found. The result then follows easily.

1			$f'(x) = g(x) + (x-p)g'(x)$ Tangent passes through $(a, (a-p)g(a))$ Equation of tangent is $y = \big(g(a) + (a-p)g'(a)\big)(x-a) + (a-p)g(a)$ (or equivalent equation) Substitution of $x = p$ into equation of tangent $y = -(a-p)^2 g'(a)$ Verification that if $g'(a) = 0$, then $y = 0$ If $y = 0$ then $g'(a) = 0$ because $a \neq p$	M1 M1 A1 E1 E1 E1 (AG) (6 marks)
	(i)		$g(x) = A(x-q)(x-r)$ identified $g'(a) = 0 \Rightarrow 2a = r + q$ (legitimately obtained) Gradient of tangent is $g(a) + (a-p)g'(a)$ $= A(a-q)(a-r)$ $= -\frac{1}{4}A(r-q)^2$	M1 A1 (AG) M1 A1 (4 marks)
	(ii)		By symmetry, the gradient of the second tangent is $-\frac{1}{4}A(p-q)^2$ (can be implied) Parallel iff $(p-q)^2 = (q-r)^2$ $\Leftrightarrow q - p = r - q$ since $p < q < r$. Tangent at $x = q$, $y = A(q-p)(q-r)(x-q)$, Meets curve again when $(q-p)(q-r)(x-q) = (x-p)(x-r)(x-q)$ $\Leftrightarrow (q-p)(q-r) = (x-p)(x-r)$ since $x \neq q$ (cancellation must be justified for M1, can be awarded if used correctly on $(x-q)^2(x-p-r+q)$ later) $\Leftrightarrow (x-q)(x-p-r+q) = 0$ $\Leftrightarrow x = p + r - q$ or $x = q$ Therefore there is only one point of intersection between the tangent and the curve if and only if $p + r - q = q$, which is if and only if the tangents are parallel. *One E mark for each direction.*	 B1 M1 A1 E1 M1 M1 M1 A1 E1 E1 (AG) (10 marks)