STEP II, 2019, Q12 MS

Part (i) requires a simple integration to calculate the values of $E(X)$ and $E(X^2)$. The required result then follows algebraically.

In part (ii), use integration to find the values of the quartiles and hence the interquartile range. Square the two values to allow them to be compared with each other.

In part (iii), the binomial expansion should be easy to write down, but note that the $(k+1)^{th}$ term is the term in x^k, not x^{k+1}. The lower quartile and median can be evaluated by integration of $f(x)$. To show the inequalities, note that $\left(\frac{1}{\mu}\right)^n = \left(1+\frac{1}{n}\right)^n$ and that each term in the expansion is positive, so the value must be greater than the sum of the first two terms. Similarly, the $(k+1)^{th}$ term of the expansion can be shown to be greater than $\frac{1}{k!}$, so the result that may be assumed will lead to the other inequality.

12	(i)		$\mu = \int_0^1 nx^n\,dx = \frac{n}{n+1}$ $\mathbb{E}(X^2) = \int_0^1 nx^{n+1}\,dx = \frac{n}{n+2}$ $\sigma^2 = \frac{n}{n+2} - \left(\frac{n}{n+1}\right)^2 = \frac{n}{(n+1)^2(n+2)}$	M1 A1 M1 M1 A1 (AG) (5 marks)
	(ii)		$\mathrm{LQ} = \frac{1}{2}, \mathrm{UQ} = \frac{\sqrt{3}}{2}$ $\mathrm{IQR} = \frac{\sqrt{3}-1}{2}$ $2\sigma = \frac{\sqrt{2}}{3}$ Squaring IQR and 2σ Comparing $\sqrt{3}$ with a rational number… …by squaring both sides Argument correct	M1 A1 B1 M1 M1 M1 A1 (7 marks)
	(iii)		$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$ $+ \frac{n(n-1)\cdots(n-k+1)}{k!}x^k + \cdots$ $\mathrm{LQ} = \left(\frac{1}{4}\right)^{1/n}$ and Median $= \left(\frac{1}{2}\right)^{1/n}$ $\left(\frac{1}{\mu}\right)^n = \left(1+\frac{1}{n}\right)^n > 1 + n\left(\frac{1}{n}\right) = 2$ So $\mu < \left(\frac{1}{2}\right)^{1/n}$ $\left(\frac{1}{\mu}\right)^n = \left(1+\frac{1}{n}\right)^n < 1 + n\left(\frac{1}{n}\right) + \frac{n^2}{2!}\left(\frac{1}{n}\right)^2 + \cdots + \frac{n^k}{k!}\left(\frac{1}{n}\right)^k + \cdots$ $< 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{k!} + \cdots$ < 4 So $\mu > \left(\frac{1}{4}\right)^{1/n}$	A1 A1 B1 M1 A1 M1 M1 A1 (8 marks)