

STEP II, 2019, Q12

- 12 The random variable X has the probability density function on the interval $[0, 1]$:

$$f(x) = \begin{cases} nx^{n-1} & 0 \leq x \leq 1, \\ 0 & \text{elsewhere,} \end{cases}$$

where n is an integer greater than 1.

- (i) Let $\mu = E(X)$. Find an expression for μ in terms of n , and show that the variance, σ^2 , of X is given by

$$\sigma^2 = \frac{n}{(n+1)^2(n+2)}.$$

- (ii) In the case $n = 2$, show without using decimal approximations that the interquartile range is less than 2σ .
- (iii) Write down the first three terms and the $(k+1)$ th term (where $0 \leq k \leq n$) of the binomial expansion of $(1+x)^n$ in ascending powers of x .

By setting $x = \frac{1}{n}$, show that μ is less than the median and greater than the lower quartile.

Note: You may assume that

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots < 4.$$



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