STEP II, 2019, Q10 MS

A diagram is very useful in this question. First, note that the triangle ABC must be isosceles and then take moments about A. In the case given in part (i) this then shows that $T > W$ and so the string will break.

In part (ii), resolve the forces vertically to find an expression for the reaction force and then this can be used to find an expression for the maximum possible value for the frictional force. W can then be eliminated using the equation in part (i) found by taking moments about A. Rearranging then leads to an expression that can be used to explain the required result.

For the third part, the values of k for which breaking and slipping occur can be found from the answers to part (i). These two values can be used to set up an inequality that must be satisfied in order for slipping to occur before the string breaks.

10	(i)		Correct diagram Moments about A: $Wa\cos\theta\,(1+2k) = 2aT\sin 2\theta$ If $2k+1 > 4\sin\theta$ then $2T\sin 2\theta > W\cos\theta\,(4\sin\theta) = 2W\sin 2\theta$ Since $\sin 2\theta > 0$, $T > W$ and so the string will break.	B2 M1 M1 A1 A1 (AG) (6 marks)
	(ii)		Resolving forces vertically: $R = ((k+1)W - T\sin\theta)$ Resolving horizontally, ring will slip if: $T\cos\theta > \mu\big((k+1)W - T\sin\theta\big)$ (= max value for friction) Moments about A: $W(2k+1) = 4T\sin\theta$ $\mu\big((k+1)W - T\sin\theta\big) = \mu\left(\frac{4(k+1)}{2k+1} - 1\right)T\sin\theta$ $\mu\left(\frac{2k+3}{2k+1}\right)T\sin\theta$ If $2k+1 > (2k+3)\mu\tan\theta$, then $\mu\left(\frac{2k+3}{2k+1}\right)\sin\theta < \cos\theta$ So the ring will slip.	 M1 M1 M1 A1 M1 A1 (AG) (6 marks)
	(iii)		Attempt to solve breaking inequality for k Breaks at $k = \frac{4\sin\theta - 1}{2}$ Attempt to solve slipping inequality for k Slips at $k = \frac{3\mu\tan\theta - 1}{2(1-\mu\tan\theta)}$ If ring slips before it breaks: $\frac{3\mu\tan\theta - 1}{2(1-\mu\tan\theta)} < \frac{4\sin\theta - 1}{2}$ (for A1, do not allow >) Confirming that inequality is being multiplied by a positive quantity. $3\mu\tan\theta - 1 < 4\sin\theta - 1$ $\mu < \frac{2\cos\theta}{2\sin\theta + 1}$	M1 A1 B1 M1 A1 E1 M1 A1 (AG) (8 marks)