STEP II, 2019, Q10

10 A small light ring is attached to the end A of a uniform rod AB of weight W and length $2a$. The ring can slide on a rough horizontal rail.

One end of a light inextensible string of length $2a$ is attached to the rod at B and the other end is attached to a point C on the rail so that the rod makes an angle of θ with the rail, where $0 < \theta < 90°$. The rod hangs in the same vertical plane as the rail.

A force of kW acts vertically downwards on the rod at B and the rod is in equilibrium.

(i) You are given that the string will break if the tension T is greater than W. Show that (assuming that the ring does not slip) the string will break if

$$2k + 1 > 4\sin\theta.$$

(ii) Show that (assuming that the string does not break) the ring will slip if

$$2k + 1 > (2k + 3)\mu\tan\theta,$$

where μ is the coefficient of friction between the rail and the ring.

(iii) You are now given that $\mu\tan\theta < 1$.

Show that, when k is increased gradually from zero, the ring will slip before the string breaks if

$$\mu < \frac{2\cos\theta}{1 + 2\sin\theta}.$$