

STEP II, 2019, Q1

- 1 Let $f(x) = (x - p)g(x)$, where g is a polynomial. Show that the tangent to the curve $y = f(x)$ at the point with $x = a$, where $a \neq p$, passes through the point $(p, 0)$ if and only if $g'(a) = 0$.

The curve C has equation

$$y = A(x - p)(x - q)(x - r),$$

where p , q and r are constants with $p < q < r$, and A is a non-zero constant.

- (i) The tangent to C at the point with $x = a$, where $a \neq p$, passes through the point $(p, 0)$. Show that $2a = q + r$ and find an expression for the gradient of this tangent in terms of A , q and r .
- (ii) The tangent to C at the point with $x = c$, where $c \neq r$, passes through the point $(r, 0)$. Show that this tangent is parallel to the tangent in part (i) if and only if the tangent to C at the point with $x = q$ does not meet the curve again.



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