STEP II, 2018, Q9 MS

Since the two particles are released from rest the distance between them will remain constant until A reaches the ground. The height of B above the ground at this point can therefore be calculated.

Application of the uniform acceleration formulae will therefore give the speeds of A and B at the moment A hits the ground and the coefficient of restitution can then be used to calculate the speed with which A rebounds. Since they both continue to move under gravity the speed of B relative to A will remain constant and therefore the time until they collide can be calculated. Once the time is known one of the uniform acceleration formulae can then be used to determine the height at which the collision happens and the two speeds at this time.

For the final part another application of the uniform acceleration formulae can be used to find the velocity of A immediately after the collision. Conservation of momentum can then be used to find the velocity of B, although care needs to be taken at this stage with the signs of the terms. Finally, the velocities before and after the collision can be used to calculate the coefficient of restitution.

When A reaches the ground for the first time B will be at a height of $9h$ above P.	**B1**
For the motion until A reaches the ground:	**M1**
$u = 0, a = g, s = 8h$	
$v^2 = u^2 + 2as$	
$v^2 = 16gh$	
Therefore $v = 4\sqrt{gh}$	**A1**
A rebounds with a speed of $2\sqrt{gh}\ ms^{-1}$	**A1**
The velocity of B relative to A for the subsequent motion will be $6\sqrt{gh}$	**B1**
The particles will therefore collide after $\frac{9h}{6\sqrt{gh}} = \frac{3h}{2\sqrt{gh}}\ s$	**M1** **A1**
For particle A:	
$u = -2\sqrt{gh}, a = g, t = \dfrac{3h}{2\sqrt{gh}}$	
$s = ut + \dfrac{1}{2}at^2 = -2\sqrt{gh}\left(\dfrac{3h}{2\sqrt{gh}}\right) + \dfrac{1}{2}g\left(\dfrac{3h}{2\sqrt{gh}}\right)^2$	**M1**
$s = -3h + \dfrac{9h}{8} = -\dfrac{15}{8}h$	**A1** **AG**
So the collision occurs a distance of $\frac{15}{8}h$ above P.	
$v = u + at = -2\sqrt{gh} + g\left(\dfrac{3h}{2\sqrt{gh}}\right)$	**M1**
$v = -\dfrac{1}{2}\sqrt{gh}$	**A1**
$u_A = \dfrac{1}{2}\sqrt{gh}$	
The velocity of B will be	**M1**
$-\dfrac{1}{2}\sqrt{gh} + 6\sqrt{gh} = \dfrac{11}{2}\sqrt{gh}$	
$u_B = \dfrac{11}{2}\sqrt{gh}$	**A1**

To hit the ground the second time with speed $4\sqrt{gh}$:

$v = 4\sqrt{gh}, a = g, s = \frac{15}{8}h$

$v^2 = u^2 + 2as$

$16gh = u^2 + \frac{15}{4}gh$ **M1**

$u^2 = \frac{49}{4}gh$ **A1**

$u = \frac{7}{2}\sqrt{gh}$ (since $u > -\frac{1}{2}\sqrt{gh}$) **E1**

Conservation of momentum for collision between the beads: **M1**

$m\left(-\frac{1}{2}\sqrt{gh}\right) + m\left(\frac{11}{2}\sqrt{gh}\right) = m\left(\frac{7}{2}\sqrt{gh}\right) + mv$

where v is the velocity of B after the collision.

$v = \frac{3}{2}\sqrt{gh}$ **A1**

$$e = \frac{\frac{7}{2}\sqrt{gh} - \frac{3}{2}\sqrt{gh}}{\frac{11}{2}\sqrt{gh} - \left(-\frac{1}{2}\sqrt{gh}\right)} = \frac{1}{3}$$

M1
A1