

## STEP II, 2018, Q8

- 8 (i) Use the substitution  $v = \sqrt{y}$  to solve the differential equation

$$\frac{dy}{dt} = \alpha y^{\frac{1}{2}} - \beta y \quad (y \geq 0, t \geq 0),$$

where  $\alpha$  and  $\beta$  are positive constants. Find the non-constant solution  $y_1(x)$  that satisfies  $y_1(0) = 0$ .

- (ii) Solve the differential equation

$$\frac{dy}{dt} = \alpha y^{\frac{2}{3}} - \beta y \quad (y \geq 0, t \geq 0),$$

where  $\alpha$  and  $\beta$  are positive constants. Find the non-constant solution  $y_2(x)$  that satisfies  $y_2(0) = 0$ .

- (iii) In the case  $\alpha = \beta$ , sketch  $y_1(x)$  and  $y_2(x)$  on the same axes, indicating clearly which is  $y_1(x)$  and which is  $y_2(x)$ . You should explain how you determined the positions of the curves relative to each other.



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