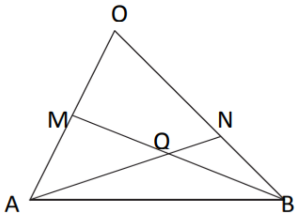
STEP II, 2018, Q7 MS

It is very useful to draw a diagram to represent the situation described at the start of this question. Defining the vectors $\boldsymbol{m}$ and $\boldsymbol{n}$ as scalar multiples of $\boldsymbol{a}$ and $\boldsymbol{b}$ (using two new unknowns) allows the position vector of Q to be written in two different ways. Since the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are not parallel, the coefficients of these vectors can be equated and this then leads to the correct expression for $\boldsymbol{m}$.

A similar process then leads to an expression for the position vector of L in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, but since L lies on OB, the coefficient of $\boldsymbol{a}$ must be 0.

It then follows that $\lambda\mu < 1$ means that L lies on the segment OB.



B1

Let k and l be such that $\boldsymbol{m} = k\boldsymbol{a}$ and $\boldsymbol{n} = l\boldsymbol{b}$

$\overrightarrow{BM} = k\boldsymbol{a} - \boldsymbol{b}$ **M1**

$$\overrightarrow{QM} = \frac{\mu}{1+\mu}(k\boldsymbol{a} - \boldsymbol{b})$$ **A1**

Similarly: **A1**

$$\overrightarrow{QN} = \frac{\nu}{1+\nu}(l\boldsymbol{b} - \boldsymbol{a})$$

Therefore: **M1**

$$\boldsymbol{q} = \overrightarrow{OM} + \overrightarrow{MQ} = k\boldsymbol{a} - \frac{\mu}{1+\mu}(k\boldsymbol{a} - \boldsymbol{b})$$

$$\boldsymbol{q} = \frac{k}{1+\mu}\boldsymbol{a} + \frac{\mu}{(1+\mu)}\boldsymbol{b}$$ **A1**

And: **M1**

$$\boldsymbol{q} = \frac{l}{1+\nu}\boldsymbol{b} + \frac{\nu}{(1+\nu)}\boldsymbol{a}$$ **A1**

Since $\boldsymbol{a}$ and $\boldsymbol{b}$ are not parallel: **M1**

$$\frac{k}{1+\mu} = \frac{\nu}{(1+\nu)}$$

Therefore **A2 AG**

$$k = \frac{(1+\mu)\nu}{1+\nu}$$

So

$$\boldsymbol{m} = \frac{(1+\mu)\nu}{1+\nu}\boldsymbol{a}$$

$$\overrightarrow{AN} = \frac{(1+\nu)\mu}{1+\mu}\boldsymbol{b} - \boldsymbol{a}$$ **B1**

Therefore: **M1**

$$\overrightarrow{OL} = \frac{(1+\mu)\nu}{1+\nu}\boldsymbol{a} + p\left(\frac{(1+\nu)\mu}{1+\mu}\boldsymbol{b} - \boldsymbol{a}\right)$$ **A1**

For some value of p

Since $\overrightarrow{OL}$ is parallel to $\boldsymbol{b}$, the coefficient of $\boldsymbol{a}$ must be 0 **M1**

$$\frac{(1+\mu)\nu}{1+\nu} - p = 0$$ **M1**

Therefore **M1**

$$\overrightarrow{OL} = \frac{(1+\mu)\nu}{1+\nu}\boldsymbol{a} + \frac{(1+\mu)\nu}{1+\nu}\left(\frac{(1+\nu)\mu}{1+\mu}\boldsymbol{b} - \boldsymbol{a}\right) = \nu\mu\boldsymbol{b}$$

So $\lambda = \mu\nu$ **A2**

$\mu\nu < 1$ means that L lies on OB. **E1**