

STEP II, 2018, Q5

5 In this question, you should ignore issues of convergence.

- (i) Write down the binomial expansion, for $|x| < 1$, of $\frac{1}{1+x}$ and deduce that

$$\ln(1+x) = -\sum_{n=1}^{\infty} \frac{(-x)^n}{n}$$

for $|x| < 1$.

- (ii) Write down the series expansion in powers of x of e^{-ax} . Use this expansion to show that

$$\int_0^{\infty} \frac{(1 - e^{-ax})e^{-x}}{x} dx = \ln(1+a) \quad (|a| < 1).$$

- (iii) Deduce the value of

$$\int_0^1 \frac{x^p - x^q}{\ln x} dx \quad (|p| < 1, |q| < 1).$$



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