

STEP II, 2018, Q3

3 (i) Let

$$f(x) = \frac{1}{1 + \tan x}$$

for $0 \leq x < \frac{1}{2}\pi$.

Show that $f'(x) = -\frac{1}{1 + \sin 2x}$ and hence find the range of $f'(x)$.

Sketch the curve $y = f(x)$.

(ii) The function $g(x)$ is continuous for $-1 \leq x \leq 1$.

Show that the curve $y = g(x)$ has rotational symmetry of order 2 about the point (a, b) on the curve if and only if

$$g(x) + g(2a - x) = 2b.$$

Given that the curve $y = g(x)$ passes through the origin and has rotational symmetry of order 2 about the origin, write down the value of

$$\int_{-1}^1 g(x) dx.$$

(iii) Show that the curve $y = \frac{1}{1 + \tan^k x}$, where k is a positive constant and $0 < x < \frac{1}{2}\pi$,

has rotational symmetry of order 2 about a certain point (which you should specify) and evaluate

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1}{1 + \tan^k x} dx.$$



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