

STEP II, 2018, Q2

- 2 A function $f(x)$ is said to be *concave* for $a < x < b$ if

$$tf(x_1) + (1 - t)f(x_2) \leq f(tx_1 + (1 - t)x_2),$$

for $a < x_1 < b$, $a < x_2 < b$ and $0 \leq t \leq 1$.

Illustrate this definition by means of a sketch, showing the chord joining the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$, in the case $x_1 < x_2$ and $f(x_1) < f(x_2)$.

Explain why a function $f(x)$ satisfying $f''(x) < 0$ for $a < x < b$ is concave for $a < x < b$.

- (i) By choosing t , x_1 and x_2 suitably, show that, if $f(x)$ is concave for $a < x < b$, then

$$f\left(\frac{u + v + w}{3}\right) \geq \frac{f(u) + f(v) + f(w)}{3},$$

for $a < u < b$, $a < v < b$ and $a < w < b$.

- (ii) Show that, if A , B and C are the angles of a triangle, then

$$\sin A + \sin B + \sin C \leq \frac{3\sqrt{3}}{2}.$$

- (iii) By considering $\ln(\sin x)$, show that, if A , B and C are the angles of a triangle, then

$$\sin A \times \sin B \times \sin C \leq \frac{3\sqrt{3}}{8}.$$



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