



STEP II, 2018, Q13 MS

The probabilities in the first part of the question can most easily be deduced by using a tree diagram.

For the second part, note that the probabilities at B must be equal to the probabilities at D by the symmetry of the problem. The sum of the four probabilities for any value of n must be equal to 1. Therefore, it is possible to deduce a recurrence relation for B_n and see that this remains at a constant value. With the values of B_n and D_n known recurrence relations for A_n and C_n can be found. These recurrence relations can be related to geometric sequences in order to find the formula for the general term.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)

| | | |
|------|--|-----------|
| (i) | $A_1 = \frac{1}{2}, C_1 = 0$ | B1 |
| | $B_1 = \frac{1}{4}, D_1 = \frac{1}{4}$ | B1 |
| | $A_2 = \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{3}{8}$ | M1 |
| | | M1 |
| | | A1 |
| | $B_2 = D_2 = \frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} = \frac{1}{4}$ | M1 |
| | | A1 |
| | $C_2 = \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{1}{8}$ | M1 |
| | | A1 |
| (ii) | $B_{n+1} = \frac{1}{2}B_n + \frac{1}{4}(A_n + C_n)$ | M1 |
| | $A_n + B_n + C_n + D_n = 1$ | M1 |
| | $B_n = D_n$ (by symmetry) | M1 |
| | Therefore $A_n + C_n = 1 - 2B_n$ | M1 |
| | $B_{n+1} = \frac{1}{4}$ and so $B_n = D_n = \frac{1}{4}$ for all n . | A1 |
| | | |
| | $A_{n+1} = \frac{1}{2}A_n + \frac{1}{4}(B_n + D_n) = \frac{1}{2}A_n + \frac{1}{8}$ | M1 |
| | | A1 |
| | $A_{n+1} - \frac{1}{4} = \frac{1}{2}\left(A_n - \frac{1}{4}\right)$ | M1 |
| | Therefore $\left(A_n - \frac{1}{4}\right)$ is a geometric sequence with common ratio $\frac{1}{2}$ | M1 |
| | $A_n = \frac{1}{4} + \left(\frac{1}{2}\right)^{n+1}$ | A1 |
| | $C_n = \frac{1}{4} - \left(\frac{1}{2}\right)^{n+1}$ | A1 |



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)