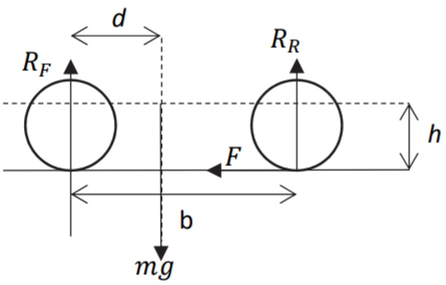
STEP II, 2018, Q11 MS

A diagram representing the situation will help to ensure that the correct calculations are performed. In particular it is important to note that the frictional force will be acting in the direction of motion of the motorbike. Taking moments about the centre of mass as instructed and then setting the reaction at the front wheel of the motorbike to 0 for the case when the front wheel loses contact with the ground gives the maximum possible frictional force for this motion. Comparing this to μR then gives the first inequality.

When the rear wheel is about to slip the frictional force will be taking its maximum value. Substituting this into the equation found by taking moments and resolving forces vertically then allows the value of this frictional force to be found. Newton's second law then gives the acceleration.

For the final part, first show that the maximum acceleration is at the moment when the front wheel would be about to leave the ground. The value of the frictional force at this point can then be found and the acceleration can then be deduced.



Taking moments about the centre of mass: **M1**

$R_F d + Fh = R_R(b - d)$ **A1**

A1

$$F = \frac{R_R(b-d) - R_F d}{h}$$ **M1** **A1**

At the time when the front wheel loses contact with the ground: **B1**

$R_F = 0$ and $R_R = mg$

$$F = \frac{mg(b-d)}{h}$$

Maximum possible frictional force is μmg **B1**

Therefore if **E1**

$$\mu mg < \frac{mg(b-d)}{h}$$ **AG**

then the rear wheel will have slipped before this point.

i.e. if

$$\mu < \frac{b-d}{h}$$

At the moment before the rear wheel slips, friction will take its maximum value **B1**

$$\frac{R_R(b-d) - R_F d}{h} = \mu R_R$$ **M1**

Resolving vertically: **M1**

$R_F + R_R = mg$

$R_R b - mgd = \mu h R_R$ **M1**

$$R_R = \frac{mgd}{b - \mu h}$$ **A1**

Therefore

$$F = \frac{\mu mgd}{b - \mu h}$$

Newton's second law: **M1**

$F = ma$

Therefore **A1**

$$a = \frac{\mu dg}{b - \mu h}$$

The front wheel would lose contact with the road when $R_F = 0$:

The acceleration is given by **E1**

$$a = \frac{R_R b - mgd}{mh}$$ **E1**

Therefore a increases as R_R increases and R_F decreases

So the maximum acceleration is at the moment when the front wheel would be about to leave the ground **E1**

At this point $F = \frac{mg(b-d)}{h}$ and so **A1**

$$a = \frac{g(b-d)}{h}$$