

STEP II, 2018, Q10

- 10 A uniform elastic string lies on a smooth horizontal table. One end of the string is attached to a fixed peg, and the other end is pulled at constant speed u . At time $t = 0$, the string is taut and its length is a . Obtain an expression for the speed, at time t , of the point on the string which is a distance x from the peg at time t .

An ant walks along the string starting at $t = 0$ at the peg. The ant walks at constant speed v along the string (so that its speed relative to the peg is the sum of v and the speed of the point on the string beneath the ant). At time t , the ant is a distance x from the peg. Write down a first order differential equation for x , and verify that

$$\frac{d}{dt} \left(\frac{x}{a + ut} \right) = \frac{v}{a + ut}.$$

Show that the time T taken for the ant to reach the end of the string is given by

$$uT = a(e^k - 1),$$

where $k = u/v$.

On reaching the end of the string, the ant turns round and walks back to the peg. Find in terms of T and k the time taken for the journey back.



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