

STEP II, 2018, Q1

- 1 Show that, if k is a root of the quartic equation

$$x^4 + ax^3 + bx^2 + ax + 1 = 0, \quad (*)$$

then k^{-1} is a root.

You are now given that a and b in (*) are both real and are such that the roots are all real.

- (i) Write down all the values of a and b for which (*) has only one distinct root.
- (ii) Given that (*) has exactly three distinct roots, show that either $b = 2a - 2$ or $b = -2a - 2$.
- (iii) Solve (*) in the case $b = 2a - 2$, giving your solutions in terms of a .

Given that a and b are both real and that the roots of (*) are all real, find necessary and sufficient conditions, in terms of a and b , for (*) to have exactly three distinct real roots.



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