

STEP II, 2017, Q7

7 The functions f and g are defined, for $x > 0$, by

$$f(x) = x^x, \quad g(x) = x^{f(x)}.$$

(i) By taking logarithms, or otherwise, show that $f(x) > x$ for $0 < x < 1$. Show further that $x < g(x) < f(x)$ for $0 < x < 1$.

Write down the corresponding results for $x > 1$.

(ii) Find the value of x for which $f'(x) = 0$.

(iii) Use the result $x \ln x \rightarrow 0$ as $x \rightarrow 0$ to find $\lim_{x \rightarrow 0} f(x)$, and write down $\lim_{x \rightarrow 0} g(x)$.

(iv) Show that $x^{-1} + \ln x \geq 1$ for $x > 0$.

Using this result, or otherwise, show that $g'(x) > 0$.

Sketch the graphs, for $x > 0$, of $y = x$, $y = f(x)$ and $y = g(x)$ on the same axes.



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