

STEP II, 2017, Q6

6 Let

$$S_n = \sum_{r=1}^n \frac{1}{\sqrt{r}},$$

where n is a positive integer.

(i) Prove by induction that

$$S_n \leq 2\sqrt{n} - 1.$$

(ii) Show that $(4k+1)\sqrt{k+1} > (4k+3)\sqrt{k}$ for $k \geq 0$.

Determine the smallest number C such that

$$S_n \geq 2\sqrt{n} + \frac{1}{2\sqrt{n}} - C.$$



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