

STEP II, 2017, Q5

- 5 A curve C is determined by the parametric equations

$$x = at^2, \quad y = 2at,$$

where $a > 0$.

- (i) Show that the normal to C at a point P , with non-zero parameter p , meets C again at a point N , with parameter n , where

$$n = -\left(p + \frac{2}{p}\right).$$

- (ii) Show that the distance $|PN|$ is given by

$$|PN|^2 = 16a^2 \frac{(p^2 + 1)^3}{p^4}$$

and that this is minimised when $p^2 = 2$.

- (iii) The point Q , with parameter q , is the point at which the circle with diameter PN cuts C again. By considering the gradients of QP and QN , show that

$$2 = p^2 - q^2 + \frac{2q}{p}.$$

Deduce that $|PN|$ is at its minimum when Q is at the origin.



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