

STEP II, 2017, Q4

4 The Schwarz inequality is

$$\left(\int_a^b f(x)g(x) dx \right)^2 \leq \left(\int_a^b (f(x))^2 dx \right) \left(\int_a^b (g(x))^2 dx \right). \quad (*)$$

(i) By setting $f(x) = 1$ in (*), and choosing $g(x)$, a and b suitably, show that for $t > 0$,

$$\frac{e^t - 1}{e^t + 1} \leq \frac{t}{2}.$$

(ii) By setting $f(x) = x$ in (*), and choosing $g(x)$ suitably, show that

$$\int_0^1 e^{-\frac{1}{2}x^2} dx \geq 12(1 - e^{-\frac{1}{4}})^2.$$

(iii) Use (*) to show that

$$\frac{64}{25\pi} \leq \int_0^{\frac{1}{2}\pi} \sqrt{\sin x} dx \leq \sqrt{\frac{\pi}{2}}.$$



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