

STEP II, 2017, Q2

- 2 The sequence of numbers x_0, x_1, x_2, \dots satisfies

$$x_{n+1} = \frac{ax_n - 1}{x_n + b}.$$

(You may assume that a, b and x_0 are such that $x_n + b \neq 0$.)

Find an expression for x_{n+2} in terms of a, b and x_n .

- (i) Show that $a + b = 0$ is a necessary condition for the sequence to be periodic with period 2.

Note: The sequence is said to be periodic with period k if $x_{n+k} = x_n$ for all n , and there is no integer m with $0 < m < k$ such that $x_{n+m} = x_n$ for all n .

- (ii) Find necessary and sufficient conditions for the sequence to have period 4.



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