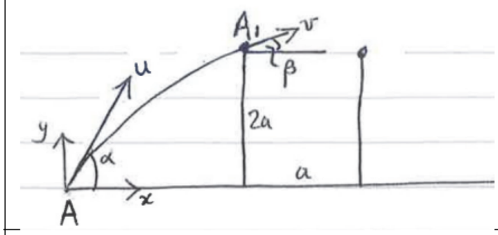
STEP II, 2017, Q11 MS

As with most mechanics questions, a large clear diagram is very useful. Although not mentioned in the question, defining the angle of projection is a very good idea in projectile questions.

Conserving energy provides a fairly standard start to this question. We then needed to transfer to kinematics to introduce angles. An important decision needs to be made about where to set the origin. It turns out that the top of the first wall makes a very sensible choice. Standard kinematic equations can be used to write the vertical and horizontal displacement when the particle passes over the second wall. Eliminating the time from these equations and using the result from the first part leads to a familiar looking trigonometric expression.

To obtain the distance of A from the foot of the wall it is useful to find the angle of projection. To do this it is useful to find something that doesn't change to form into an equation; in this case the horizontal component of the velocity. This leads to finding the cosine of the angle of projection. Using a trigonometric identity can turn it into the sine of the angle. You can then use a kinematics equation to describe the vertical displacement, finding a quadratic equation for the time taken to get to the height of the wall. This time can be used to find the horizontal displacement.

Part (ii) follows a similar pattern to part (i). Energy considerations can be used to find the speed over the first wall. Then kinematics equations (or more directly the trajectory equation) can used to form a quadratic equation in the tan of the angle passing over the first wall if it just passes the second wall. Examining the discriminant (after a fair amount of algebra) shows that this equation does not have a solution, so the particle cannot pass over the second wall.

		
(i)		
At A, $KE = \frac{1}{2}mu^2 = \frac{5}{2}mag, PE = 0$	**B1**	
At $A_1, K = \frac{1}{2}mv^2, PE = 2mag$	**B1**	
Conservation of energy: $$\frac{5}{2}mag = \frac{1}{2}mv^2 + 2mag$$	**M1**	
$v^2 = ga$		
$v = \sqrt{ga}$	**A1**	
	[4]	
If angle at A_1 is β and it just passes the second wall then we have:		
$0 = v\sin\theta\, t - \frac{1}{2}gt^2$	**M1**	Using $s = ut + \frac{1}{2}at^2$
So $t = \frac{2v}{g}\sin\beta$	**A1**	Solving for t at second wall.
Also, $a = v\cos\beta\, t$	**M1**	Considering horizontal distance
$= \frac{2v^2\sin\beta\cos\beta}{g}$		N.b. Some candidates may just quote this (or equivalent). Give full credit.
$= 2a\sin\beta\cos\beta$	**A1**	Combining previous results.
So $\sin(2\beta) = 1$	**A1**	
Therefore $\beta = 45°$	**AG**	Condone absence of domain considerations.
	[5]	
x velocity is constant so		
$u\cos\alpha = v cos\,\beta$	**M1**	Comparing x velocities
$\sqrt{5ag}\cos\alpha = \sqrt{ag}\frac{1}{\sqrt{2}}$ $\cos\alpha = \frac{1}{\sqrt{10}}$	**A1**	
$\sin\alpha = \frac{3}{\sqrt{10}}, \tan\alpha = 3$	**A1**	Converting to a more useful ratio.

Method 1: $$2a = \sqrt{5ag}\frac{3}{\sqrt{10}}t - \frac{1}{2}gt^2$$ $$= \frac{3\sqrt{ag}}{\sqrt{2}}t - \frac{1}{2}gt^2$$ So $$t^2 - \frac{3\sqrt{2a}}{\sqrt{g}}t + \frac{4a}{g} = 0$$	**M1**	Using $s = ut + \frac{1}{2}at^2$
$$\left(t - \sqrt{\frac{2a}{g}}\right)\left(t - 2\sqrt{\frac{2a}{g}}\right) = 0$$		
First time over the wall means that $t = \sqrt{\frac{2a}{g}}$	**A1**	
So $d = ucos\,\theta\, t = \sqrt{5ag} \times \frac{1}{\sqrt{10}} \times \sqrt{\frac{2a}{g}} = a$	**A1**	
Method 2: $$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$$	**M1**	Using trajectory equation
$$2a = 3x - \frac{x^2}{a}$$	**A1**	Combining with previous results
$$(x-a)(x-2a) = 0$$		
$$x = a$$	**A1**	
	[6]	
If the speed at h above first wall is v then by conserving energy, $$\frac{1}{2}5ag = \frac{1}{2}v^2 + (2a+h)g$$	**M1**	
$$v^2 = ag - 2gh$$	**B1**	
Using trajectory equation with origin at top of first wall and angle β as particle moves over first wall: $$y = h + x\tan\beta - \frac{gx^2(1+\tan^2\beta)}{2v^2}$$ When $x = a$ we need $y = 0$: $$0 = h + a\tan\beta - \frac{ga^2(1+\tan^2\beta)}{2v^2}$$	**M1**	Use of trajectory equation (might be several kinematics equations effectively leading to the same thing)
Treating this as a quadratic in $\tan\beta$: $$-\frac{ga^2}{2v^2}\tan^2\beta + a\tan\beta + h - \frac{ga^2}{2v^2} = 0$$ $$-ga^2\tan^2\beta + 2av^2\tan\beta + 2hv^2 - ga^2 = 0$$ The discriminant is: $$4a^2v^4 + 4ga^2(2hv^2 - ga^2)$$	**M1**	Considering the quadratic (or equivalently differentiating to find the max)
$$= 4a^2(g^2(a^2 - 4ah + 4h^2) + 2g^2h(a-2h) - g^2a^2))$$ $$= 4a^2g^2(a^2 - 4ah + 4h^2 + 2ah - 4h^2 - a^2)$$ $$= -8a^3g^2h$$ $$< 0$$ Therefore no solution.	**A1**	Obtaining a clearly negative discriminant – this might take many alternative forms.
	[5]	