STEP II, 2017, Q10 MS

With questions involving lots of "show that" work it is particularly important to not simply write down expressions which are true, but could have come from "working backwards".

The first critical idea is that the work done by the car is $\int_0^d F \, \mathrm{d}x$ where F is the force exerted by the car.

The second critical idea is to use Newton's 2nd Law:

$$ma = F - (Av^2 + R)$$

To obtain the second integral a change of variable is required. This needed a clear explanation of how to change both the limits and the $\mathrm{d}x$.

In part (i) the integral had to be split up to reflect the two parts of the journey. In theory either the integral with respect to x or with respect to v could be used, but you might think that the fact that you were led towards the integral with respect to v just above suggests that it would be the better choice, and this is indeed the case. It was also important to explain why the $R > ma$ condition was needed.

In part (ii) the given condition needs to be interpreted in terms of the speed at which the force is zero. This needs to be compared to w to check that it is achieved. Then some more integrals similar to part (i) and some algebra leads to the required result.

$ma = F - (Av^2 + R)$ $\text{WD} = \int_0^d F\,\text{d}x$ $= \int_0^d \left(ma + Av^2 + R\right)\text{d}x$ Since $a = v\dfrac{\text{d}v}{\text{d}x}$ $\text{WD} = \int_{x=0}^{x=d} \left(ma + Av^2 + R\right)\dfrac{\text{d}x}{\text{d}v}\,\text{d}v$ $= \int_{x=0}^{x=d} \left(ma + Av^2 + R\right)\dfrac{v}{a}\,\text{d}v$ Using $v^2 = u^2 + 2as$ with $v = w, u = 0, s = d \Rightarrow$ $w = \sqrt{2ad}$ Therefore: $\text{WD} = \int_{v=0}^{v=w} \dfrac{\left(ma + Av^2 + R\right)v}{a}\,\text{d}v$	B1 M1 AG B1 M1 B1 AG [5]	Clear use of N2L Attempting to change variable of integration. Justifying limits. Ignore absence of $\pm$
(i) $\text{WD} = \left[\left(m + \dfrac{R}{a}\right)\dfrac{v^2}{2} + \dfrac{Av^4}{4a}\right]_0^{\sqrt{2ad}}$ $= \left(m + \dfrac{R}{a}\right)ad + Aad^2$ For second half-journey, $\text{WD} = \int_w^0 \dfrac{\left(-ma + Av^2 + R\right)v}{-a}\,\text{d}v$ $= -mad + Rd + Aad^2$ Summing gives $2dR + 2Aad^2$ $R > ma \Rightarrow F = Av^2 + R - ma > 0$ always	M1 A1 B1B1 A1 AG E1 [6]	Performing integration Correct answer in terms of d. B1 for correct limits B1 for correct integrand N.b. integrals may be combined to get to the same result.

(ii) If $R < ma$ then F is zero when $Av^2 = ma - R$ i.e. when $v = V = \sqrt{\dfrac{ma-R}{A}}$	B1	Finding an expression for the critical speed.
For F to fall to zero during motion, $V < w$ i.e. when $\dfrac{ma-R}{A} < 2ad$ i.e. $R > ma - 2Aad$	E1 E1	
In this case, $\text{WD} = mad + Rd + Aad^2$, as before, for the first half-journey	B1	
For the second half $\quad \text{WD} = \int_w^V \dfrac{(-ma + Av^2 + R)v}{-a}\,dv$	M2	
$\left[(ma-R)\dfrac{v^2}{2a} - \dfrac{Av^4}{4a}\right]_w^V$	A1	
$= \dfrac{1}{2a}(ma-R)\left(\dfrac{ma-R}{A}\right) - \dfrac{A}{4a}\left(\dfrac{ma-R}{A}\right)^2 -$ $\dfrac{1}{2a}(ma-R)(2ad) + \dfrac{A}{4a}(4a^2d^2)$ $= \dfrac{1}{2Aa}(ma-R)^2 - \dfrac{1}{4Aa}(ma-R)^2 - (ma-R)d +$ Aad^2	M1	Substituting expressions for V and w.
$= \dfrac{1}{4Aa}(ma-R)^2 - mad + Rd + Aad^2$	A1 CAO	Without wrong working
So total $\text{WD} = \dfrac{1}{4Aa}(ma-R)^2 + 2Rd + 2Aad^2$	AG [9]	