STEP II, 2017, Q10

10 A car of mass m makes a journey of distance $2d$ in a straight line. It experiences air resistance and rolling resistance so that the total resistance to motion when it is moving with speed v is $Av^2 + R$, where A and R are constants.

The car starts from rest and moves with constant acceleration a for a distance d. Show that the work done by the engine for this half of the journey is

$$\int_0^d (ma + R + Av^2)\,\mathrm{d}x$$

and that it can be written in the form

$$\int_0^w \frac{(ma + R + Av^2)v}{a}\,\mathrm{d}v\,,$$

where $w = \sqrt{2ad}$.

For the second half of the journey, the acceleration of the car is $-a$.

(i) In the case $R > ma$, show that the work done by the engine for the whole journey is

$$2Aad^2 + 2Rd\,.$$

(ii) In the case $ma - 2Aad < R < ma$, show that at a certain speed the driving force required to maintain the constant acceleration falls to zero.

Thereafter, the engine does no work (and the driver applies the brakes to maintain the constant acceleration). Show that the work done by the engine for the whole journey is

$$2Aad^2 + 2Rd + \frac{(ma - R)^2}{4Aa}\,.$$