

STEP II, 2017, Q1

- 1 **Note:** In this question you may use without proof the result $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$.

Let

$$I_n = \int_0^1 x^n \arctan x \, dx,$$

where $n = 0, 1, 2, 3, \dots$.

- (i) Show that, for $n \geq 0$,

$$(n+1)I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2} \, dx$$

and evaluate I_0 .

- (ii) Find an expression, in terms of n , for $(n+3)I_{n+2} + (n+1)I_n$.
Use this result to evaluate I_4 .

- (iii) Prove by induction that, for $n \geq 1$,

$$(4n+1)I_{4n} = A - \frac{1}{2} \sum_{r=1}^{2n} (-1)^r \frac{1}{r},$$

where A is a constant to be determined.



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