STEP II, 2016, Q9 MS

Question 9

The result in part (i) follows from consideration of kinetic energy lost and work done.

In part (ii) apply conservation of momentum to the combined block and bullet after the bullet hits the block. By comparing to the case in part (i) the motion of the bullet until it is at rest relative to the block can be analysed. Once all of the relevant equations of motion have been written down, a series of simultaneous equations will have been found from which the values of b and c can be found.

(i)	Kinetic energy lost by bullet is $\frac{1}{2}mu^2$	M1
	Work done against resistances is Ra	M1
	Energy lost = Work done	M1
	Therefore $a = \frac{mu^2}{2R}$.	A1
(ii)	Let v be the velocity of the combined block and bullet once the bullet has stopped moving relative to the block. Momentum is conserved, so $mu = (M+m)v$	M1 A1
	In the case where the block was stationary, the bullet comes to rest over a distance of a, so its acceleration is $-\frac{u^2}{2a}$.	M1 A1
	Consider the motion of the bullet until it comes to rest relative to the block: $$v^2 = u^2 + 2\left(-\frac{u^2}{2a}\right)(b+c)$$	M1 A1
	Since $v = \frac{mu}{M+m}$: $$\left(\frac{mu}{M+m}\right)^2 = u^2 - \frac{u^2}{a}(b+c)$$	M1
	And so: $$a\left(\frac{m}{M+m}\right)^2 = a - b - c$$	A1
	The acceleration of the block must be $\frac{m}{M}$ times the acceleration of the bullet in the case where the block was fixed.	M1
	Therefore, the block accelerates from rest to a speed of $\frac{mu}{M+m}$ over a distance of c.	M1
	$'v^2 = u^2 + 2as'$: $$\left(\frac{mu}{M+m}\right)^2 = 0 + \frac{mu^2c}{Ma}$$	M1 A1
	Therefore: $$\left(\frac{m}{M+m}\right)^2 = \frac{mc}{Ma}$$ and so $$c = \frac{mMa}{(M+m)^2}$$	A1
	Substituting to get b: $$a\left(\frac{m}{M+m}\right)^2 = a - b - \frac{mMa}{(M+m)^2}$$	M1
	$$b = a\left(1 - \frac{mM}{(M+m)^2} - \frac{m^2}{(M+m)^2}\right)$$	M1
	$$b = \frac{Ma}{(M+m)^2}$$	A1