

STEP II, 2016, Q8

- 8 Evaluate the integral

$$\int_{m-\frac{1}{2}}^{\infty} \frac{1}{x^2} dx \quad (m > \frac{1}{2}).$$

Show by means of a sketch that

$$\sum_{r=m}^n \frac{1}{r^2} \approx \int_{m-\frac{1}{2}}^{n+\frac{1}{2}} \frac{1}{x^2} dx, \quad (*)$$

where m and n are positive integers with $m < n$.

- (i) You are given that the infinite series $\sum_{r=1}^{\infty} \frac{1}{r^2}$ converges to a value denoted by E . Use (*) to obtain the following approximations for E :

$$E \approx 2; \quad E \approx \frac{5}{3}; \quad E \approx \frac{33}{20}.$$

- (ii) Show that, when r is large, the error in approximating $\frac{1}{r^2}$ by $\int_{r-\frac{1}{2}}^{r+\frac{1}{2}} \frac{1}{x^2} dx$ is approximately $\frac{1}{4r^4}$.

Given that $E \approx 1.645$, show that $\sum_{r=1}^{\infty} \frac{1}{r^4} \approx 1.08$.



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