

## STEP II, 2016, Q6

- 6 This question concerns solutions of the differential equation

$$(1 - x^2) \left( \frac{dy}{dx} \right)^2 + k^2 y^2 = k^2 \quad (*)$$

where  $k$  is a positive integer.

For each value of  $k$ , let  $y_k(x)$  be the solution of  $(*)$  that satisfies  $y_k(1) = 1$ ; you may assume that there is only one such solution for each value of  $k$ .

- (i) Write down the differential equation satisfied by  $y_1(x)$  and verify that  $y_1(x) = x$ .
- (ii) Write down the differential equation satisfied by  $y_2(x)$  and verify that  $y_2(x) = 2x^2 - 1$ .
- (iii) Let  $z(x) = 2(y_n(x))^2 - 1$ . Show that

$$(1 - x^2) \left( \frac{dz}{dx} \right)^2 + 4n^2 z^2 = 4n^2$$

and hence obtain an expression for  $y_{2n}(x)$  in terms of  $y_n(x)$ .

- (iv) Let  $v(x) = y_n(y_m(x))$ . Show that  $v(x) = y_{mn}(x)$ .



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