

## STEP II, 2016, Q5

5 In this question, the definition of  $\binom{p}{q}$  is taken to be

$$\binom{p}{q} = \begin{cases} \frac{p!}{q!(p-q)!} & \text{if } p \geq q \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Write down the coefficient of  $x^n$  in the binomial expansion for  $(1-x)^{-N}$ , where  $N$  is a positive integer, and write down the expansion using the  $\Sigma$  summation notation.

By considering  $(1-x)^{-1}(1-x)^{-N}$ , where  $N$  is a positive integer, show that

$$\sum_{j=0}^n \binom{N+j-1}{j} = \binom{N+n}{n}.$$

(ii) Show that, for any positive integers  $m$ ,  $n$  and  $r$  with  $r \leq m+n$ ,

$$\binom{m+n}{r} = \sum_{j=0}^r \binom{m}{j} \binom{n}{r-j}.$$

(iii) Show that, for any positive integers  $m$  and  $N$ ,

$$\sum_{j=0}^n (-1)^j \binom{N+m}{n-j} \binom{m+j-1}{j} = \binom{N}{n}.$$



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)