

STEP II, 2016, Q4

4 Let

$$y = \frac{x^2 + x \sin \theta + 1}{x^2 + x \cos \theta + 1}.$$

(i) Given that x is real, show that

$$(y \cos \theta - \sin \theta)^2 \geq 4(y - 1)^2.$$

Deduce that

$$y^2 + 1 \geq 4(y - 1)^2,$$

and hence that

$$\frac{4 - \sqrt{7}}{3} \leq y \leq \frac{4 + \sqrt{7}}{3}.$$

(ii) In the case $y = \frac{4 + \sqrt{7}}{3}$, show that

$$\sqrt{y^2 + 1} = 2(y - 1)$$

and find the corresponding values of x and $\tan \theta$.



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