

STEP II, 2016, Q3

3 For each non-negative integer n , the polynomial f_n is defined by

$$f_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}.$$

- (i) Show that $f'_n(x) = f_{n-1}(x)$ (for $n \geq 1$).
- (ii) Show that, if a is a real root of the equation

$$f_n(x) = 0, \tag{*}$$

then $a < 0$.

- (iii) Let a and b be distinct real roots of (*), for $n \geq 2$. Show that $f'_n(a)f'_n(b) > 0$ and use a sketch to deduce that $f_n(c) = 0$ for some number c between a and b .

Deduce that (*) has at most one real root. How many real roots does (*) have if n is odd? How many real roots does (*) have if n is even?



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