STEP II, 2016, Q13 MS

Question 13

For part (i) the approximation of the binomial distribution by a normal distribution should be known and the area under the curve (applying a continuity correction) can then be approximated by a rectangle.

The second result follows from a similar approximation and the use of the formula for a probability from the binomial distribution.

Part (iii) follows from an approximation of a Poisson distribution with a normal distribution and again approximating the required area by a rectangle.

(i)	$X \sim B(16, \frac{1}{2})$ is approximated by $Y \sim N(8,4)$, so $P(X = 8) \approx P(\frac{15}{2} < Y < \frac{17}{2})$	B1 B1
	In terms of $Z \sim N(0,1)$, this is $P(-\frac{1}{4} < Z < \frac{1}{4})$	A1
	The probability is therefore given by $$\int_{-\frac{1}{4}}^{\frac{1}{4}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$	M1
	This can be approximated as a rectangle with a width of $\frac{1}{2}$ and a height of $\frac{1}{\sqrt{2\pi}}$. The area is therefore $\frac{1}{2\sqrt{2\pi}}$ $$P(X = 8) \approx \frac{1}{2\sqrt{2\pi}}$$	M1 A1
(ii)	$X \sim B(2n, \frac{1}{2})$ can be approximated by $Y \sim N(n, \frac{n}{2})$, so $P(X = n) \approx P(\frac{2n-1}{2} < Y < \frac{2n+1}{2})$	B1 B1
	In the same way as part (i) $P(X = n)$ can be approximated by a rectangle of height $\frac{1}{\sqrt{2\pi}}$. The width will now be $\sqrt{\frac{2}{n}}$.	M1 A1
	Therefore: $$P(X = n) = \frac{(2n)!}{n!\,n!}\left(\frac{1}{2}\right)^{2n} \approx \frac{1}{\sqrt{n\pi}}$$	M1 A1 A1
	Rearranging gives: $$(2n)! \approx \frac{2^{2n}(n!)^2}{\sqrt{n\pi}} \qquad (*)$$	B1
(iii)	$X \sim Po(n)$ can be approximated by $Y \sim N(n, n)$, so $P(X = n) \approx P(\frac{2n-1}{2} < Y < \frac{2n+1}{2})$	B1
	In the same way as part (i) $P(X = n)$ can be approximated by a rectangle of height $\frac{1}{\sqrt{2\pi}}$. The width will now be $\sqrt{\frac{1}{n}}$. The area is therefore $\frac{1}{\sqrt{2\pi n}}$.	M1 A1
	Therefore:	
	$$\frac{e^{-n}n^n}{n!} \approx \frac{1}{\sqrt{2\pi n}}$$	M1 A1
	Which simplifies to: $$n! \approx \sqrt{2\pi n}e^{-n}n^n$$	A1