

STEP II, 2016, Q12

12 Starting with the result $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

Write down, without proof, the corresponding result for four events A , B , C and D .

A pack of n cards, numbered $1, 2, \dots, n$, is shuffled and laid out in a row. The result of the shuffle is that each card is equally likely to be in any position in the row. Let E_i be the event that the card bearing the number i is in the i th position in the row. Write down the following probabilities:

- (i) $P(E_i)$;
- (ii) $P(E_i \cap E_j)$, where $i \neq j$;
- (iii) $P(E_i \cap E_j \cap E_k)$, where $i \neq j$, $j \neq k$ and $k \neq i$.

Hence show that the probability that at least one card is in the same position as the number it bears is

$$1 - \frac{1}{2!} + \frac{1}{3!} - \dots + (-1)^{n+1} \frac{1}{n!}.$$

Find the probability that exactly one card is in the same position as the number it bears.



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