STEP II, 2016, Q11 MS

Question 11

The particles must collide if they would be in the same position for one particular value of t. Therefore, writing out the equations of motion for the two particles and eliminating the variables that are not needed the required result can be reached.

For the second part, the time of the collision can be found by considering the heights of the bullet and target at time t and noting that these must be equal. Once the value of t has been found, the fact that this must be positive leads to the inequality that is required for the first result.

For the final part, note that gravity affects both the bullet and target in the same way, so if it is ignored then the time of collision (if there is one) will be the same and this is a situation as in part (i). Clearly, in part (i) the two objects must be moving towards each other if there is to be a collision.

(i)	Since the particles collide there is a value of t such that $$a + ut\cos\alpha = vt\cos\beta$$ $$ut\sin\alpha = b + vt\sin\beta$$	**M1**
	Multiply the first equation by b and make ab the subject: $$ab = bvt\cos\beta - but\cos\alpha$$	**M1**
	Multiply the second equation by a and make ab the subject: $$ab = aut\sin\alpha - avt\sin\beta$$	**M1**
	Equating: $$bvt\cos\beta - but\cos\alpha = aut\sin\alpha - avt\sin\beta$$ and so: $$aut\sin\alpha + but\cos\alpha = bvt\cos\beta + avt\sin\beta$$	**M1**
	$$aut\sin\alpha + but\cos\alpha = R_1\sin(\alpha + \theta_1)$$ where $R_1^2 = (aut)^2 + (but)^2$ and $\tan\theta_1 = \frac{b}{a}$	**M1** **A1** **A1**
	$$bvt\cos\beta + avt\sin\beta = R_2\sin(\beta + \theta_2)$$ where $R_2^2 = (avt)^2 + (bvt)^2$ and $\tan\theta_2 = \frac{b}{a}$	**M1** **A1** **A1**
	Since $\theta_1 = \theta_2$: $$R_1\sin(\theta + \alpha) = R_2\sin(\theta + \beta)$$ and since $vR_1 = uR_2$: $$u\sin(\theta + \alpha) = v\sin(\theta + \beta) \qquad (*)$$	**M1** **A1**
(ii)	Vertically: Bullet's height above the ground at time t is $b + vt\sin\beta - \frac{1}{2}gt^2$ Target's height above the ground at time t is $ut\sin\alpha - \frac{1}{2}gt^2$ Therefore the collision must occur when $t = \frac{b}{u\sin\alpha - v\sin\beta}$	**M1** **M1** **A1**
	The vertical height of the target at this time is $\frac{bu\sin\alpha}{u\sin\alpha - v\sin\beta} - \frac{1}{2}g\left(\frac{b}{u\sin\alpha - v\sin\beta}\right)^2$	**A1**
	If this is before it reaches the ground: $$\frac{bu\sin\alpha}{u\sin\alpha - v\sin\beta} - \frac{1}{2}g\left(\frac{b}{u\sin\alpha - v\sin\beta}\right)^2 > 0$$	**M1**
	Therefore: $$2bu\sin\alpha\,(u\sin\alpha - v\sin\beta) - b^2g > 0$$	
	$$2u\sin\alpha\,(u\sin\alpha - v\sin\beta) > bg$$	**A1**
	Both the bullet and target are affected equally by gravity, so any collision would correspond to the time for the straight line motion in part (i)	**B1**
	In part (i) there can clearly only be a collision if $\alpha > \beta$	**B1**