

STEP II, 2016, Q11

- 11 (i) Two particles move on a smooth horizontal surface. The positions, in Cartesian coordinates, of the particles at time t are $(a + ut \cos \alpha, ut \sin \alpha)$ and $(vt \cos \beta, b + vt \sin \beta)$, where a, b, u and v are positive constants, α and β are constant acute angles, and $t \geq 0$.

Given that the two particles collide, show that

$$u \sin(\theta + \alpha) = v \sin(\theta + \beta),$$

where θ is the acute angle satisfying $\tan \theta = \frac{b}{a}$.

- (ii) A gun is placed on the top of a vertical tower of height b which stands on horizontal ground. The gun fires a bullet with speed v and (acute) angle of elevation β . Simultaneously, a target is projected from a point on the ground a horizontal distance a from the foot of the tower. The target is projected with speed u and (acute) angle of elevation α , in a direction directly away from the tower.

Given that the target is hit before it reaches the ground, show that

$$2u \sin \alpha (u \sin \alpha - v \sin \beta) > bg.$$

Explain, with reference to part (i), why the target can only be hit if $\alpha > \beta$.



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