STEP II, 2016, Q10 MS

Question 10

The first requirement will be to find the centre of mass of the triangle. Once this is done a diagram will be very useful and notations will need to be added for various distances, angles and the frictional force. From this diagram the forces can be resolved in two perpendicular directions and moments can be taken. This leads to a series of equations which can be solved to work out the value that the frictional force would have to take to prevent slipping. From this the required result can be established.

	Find the centre of mass of the triangle: Let the two sides of the triangle with equal length have length b and the other side have length $2a$. Let $\bar{x}$ be the distance of the centre of mass from the side BC and along the line of symmetry.	**M1**
	$(2a+2b)\bar{x} = 2b\left(\frac{1}{2}b\cos\theta\right)$	**M1** **M1**
	$\bar{x} = \frac{b^2\cos\theta}{2(a+b)}$	**A1**
	Let the point of contact between the triangle and the peg be a distance y from the midpoint of BC. Let the weight of the triangle be W, the reaction force at the peg be R and the frictional force at the peg be F. Let the angle between BC and the horizontal be α.	**B1** **B1** **B1**
	Resolving parallel to BC: $F = W\sin\alpha$	**M1** **A1**
	Resolving perpendicular to BC: $R = W\cos\alpha$	**M1** **A1**
	$\tan\alpha = \frac{y}{\bar{x}}$	**B1**
	To prevent slipping: $F \le \mu R$	**M1**
	$\mu \ge \tan\alpha$	**A1**
	Therefore $\mu \ge \frac{2y(a+b)}{b^2\cos\theta}$	**M1** **A1**
	and y can take any value up to a, so the limit on μ is when $y = a$.	**M1**
	$\mu \ge \frac{2a(a+b)}{b^2\cos\theta}$	
	Since $a = b\sin\theta$:	**M1**
	$\mu \ge \frac{2\sin\theta(\sin\theta+1)}{\cos\theta} = 2\tan\theta(1+\sin\theta)$	**M1** **A1**