STEP II, 2015, Q9 MS

Question 9

A diagram to represent this situation will show the angles that will be required to calculate the moments of each of the particles about A in terms of θ. Following this, simple trigonometric manipulation should lead to a relationship between $\sin\theta$ and $\cos\theta$. From this, either a right-angled triangle or one of the basic trigonometric identities can be used to reach the required result.

For the second part of the question the amount of potential energy that needs to be gained by the system should be easy to calculate and this must be equal to the initial kinetic energy of the system.

Question 9

(i)	Taking moments about A:	
	$M_B = 3mga\sin(30+\theta)$	**M1 A1**
	$M_C = 5mga\sin(30-\theta)$	**M1 A1**
	$M_B = M_C$	**B1**
	$5mga(\cos 30\sin\theta + \cos\theta\sin 30) = 3mga(\cos 30\sin\theta - \cos\theta\sin 30)$	**M1**
	$5\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right) = 3\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$	**A1**
	Therefore $4\sqrt{3}\sin\theta = \cos\theta$	**A1**
	Either Use $\sin^2\theta + \cos^2\theta \equiv 1$ and justify choice of positive square root. **Or** Draw right angled triangle such that $\tan\theta = \frac{1}{4\sqrt{3}}$ and calculate the length of the hypotenuse.	**M1**
	$\sin\theta = \frac{1}{7}$	**A1**
(ii)	Let h_1 be the vertical distance of B below A. Let h_2 be the vertical distance of C below A.	
	$h_1 = a\sin\left(\frac{\pi}{3} - \theta\right) = \frac{11}{14}a$	**M1 M1 A1**
	$h_2 = a\sin\left(\frac{\pi}{3} + \theta\right) = \frac{13}{14}a$	**M1 A1**
	If X is the centre of mass of the triangle: $AX = h = \frac{3h_1+5h_2}{8} = \frac{7}{8}a$	**M1 A1**
	Conservation of energy: $4mv^2 \geq 8mg.2h$ for complete revolutions.	**M1 A1**
	Therefore $v_0 = \sqrt{\frac{7ga}{2}}$.	**A1**

Question 9 Alternative part (i)

(i)	Let X be the centre of mass of the triangle and let the distance CX be d.	
	Taking moments about X: $5mgd\cos\theta = 3mg(a-d)\cos\theta$	**M1 A1**
	Therefore $5d = 3(a-d)$, so $d = \frac{3}{8}a$.	**A1**
	X must lie on BC and $\angle XAC = 30-\theta$.	**B1**
	$\sin(30-\theta) = \dfrac{\frac{3}{8}a\cos\theta}{a}$	**M1**
	$\sin 30\cos\theta + \cos 30\sin\theta = \frac{3}{8}\cos\theta$	**M1**
	$\frac{\cos\theta}{8} = \frac{\sqrt{3}\sin\theta}{2}$.	**A1**
	Therefore $\cos\theta = 4\sqrt{3}\sin\theta$ and so $\cos^2\theta = 48\sin^2\theta$	**M1**
	$\sin^2\theta = \frac{1}{49}$, and so (since θ is acute) $\sin\theta = \frac{1}{7}$.	**M1 A1**