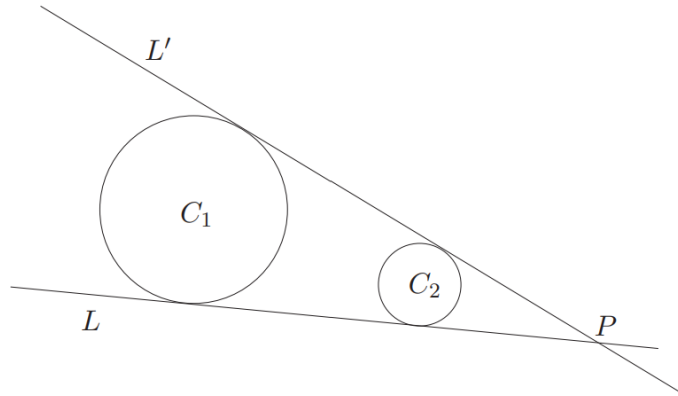


STEP II, 2015, Q8

8



The diagram above shows two non-overlapping circles C_1 and C_2 of different sizes. The lines L and L' are the two common tangents to C_1 and C_2 such that the two circles lie on the same side of each of the tangents. The lines L and L' intersect at the point P which is called the *focus* of C_1 and C_2 .

- (i) Let \mathbf{x}_1 and \mathbf{x}_2 be the position vectors of the centres of C_1 and C_2 , respectively. Show that the position vector of P is

$$\frac{r_1 \mathbf{x}_2 - r_2 \mathbf{x}_1}{r_1 - r_2},$$

where r_1 and r_2 are the radii of C_1 and C_2 , respectively.

- (ii) The circle C_3 does not overlap either C_1 or C_2 and its radius, r_3 , satisfies $r_1 \neq r_3 \neq r_2$. The focus of C_1 and C_3 is Q , and the focus of C_2 and C_3 is R . Show that P , Q and R lie on the same straight line.
- (iii) Find a condition on r_1 , r_2 and r_3 for Q to lie half-way between P and R .



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com