

## STEP II, 2015, Q7

7 A circle  $C$  is said to be *bisected* by a curve  $X$  if  $X$  meets  $C$  in exactly two points and these points are diametrically opposite each other on  $C$ .

(i) Let  $C$  be the circle of radius  $a$  in the  $x$ - $y$  plane with centre at the origin. Show, by giving its equation, that it is possible to find a circle of given radius  $r$  that bisects  $C$  provided  $r > a$ . Show that no circle of radius  $r$  bisects  $C$  if  $r \leq a$ .

(ii) Let  $C_1$  and  $C_2$  be circles with centres at  $(-d, 0)$  and  $(d, 0)$  and radii  $a_1$  and  $a_2$ , respectively, where  $d > a_1$  and  $d > a_2$ . Let  $D$  be a circle of radius  $r$  that bisects both  $C_1$  and  $C_2$ . Show that the  $x$ -coordinate of the centre of  $D$  is  $\frac{a_2^2 - a_1^2}{4d}$ .

Obtain an expression in terms of  $d$ ,  $r$ ,  $a_1$  and  $a_2$  for the  $y$ -coordinate of the centre of  $D$ , and deduce that  $r$  must satisfy

$$16r^2d^2 \geq (4d^2 + (a_2 - a_1)^2) (4d^2 + (a_2 + a_1)^2).$$



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