

STEP II, 2015, Q5

5 In this question, the arctan function satisfies $0 \leq \arctan x < \frac{1}{2}\pi$ for $x \geq 0$.

(i) Let

$$S_n = \sum_{m=1}^n \arctan \left(\frac{1}{2m^2} \right),$$

for $n = 1, 2, 3, \dots$. Prove by induction that

$$\tan S_n = \frac{n}{n+1}.$$

Prove also that

$$S_n = \arctan \frac{n}{n+1}.$$

(ii) In a triangle ABC , the lengths of the sides AB and BC are $4n^2$ and $4n^4 - 1$, respectively, and the angle at B is a right angle. Let angle $BCA = 2\alpha_n$. Show that

$$\sum_{n=1}^{\infty} \alpha_n = \frac{1}{4}\pi.$$



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