STEP II, 2015, Q4 MS

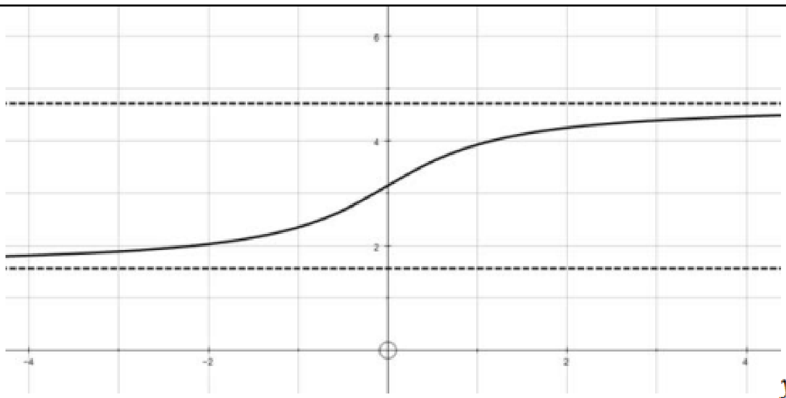
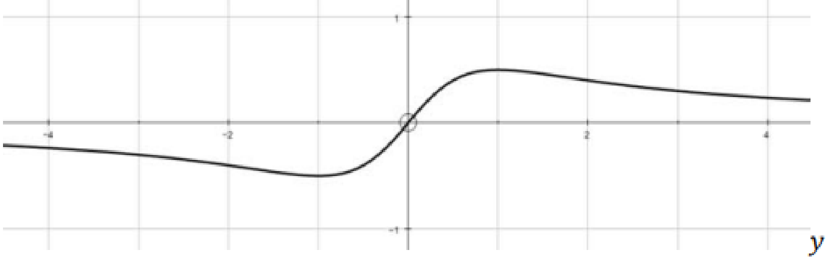
Question 4

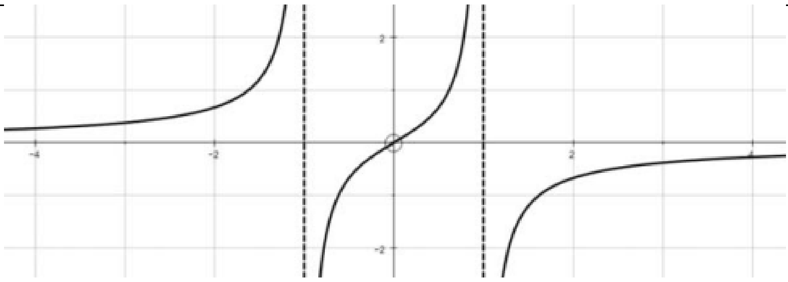
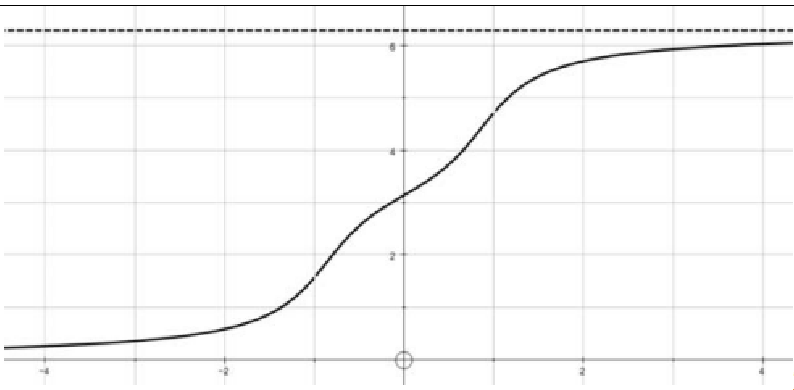
For the first part, note that the graph of $\arctan x$ satisfies the requirement of being continuous, but does not satisfy $f(0) = \pi$. Since $\tan(x + \pi) = \tan x$, a translation of the graph of $y = \arctan x$ vertically by a distance of π gives the required graph.

It should be clear that the graph of $y = \frac{x}{1+x^2}$ has no vertical asymptotes, approaches the x-axis as $x \to \pm\infty$ and passes through the origin. Identifying the stationary points should be the next task after which a graph should be easy to sketch. The graph of $y = g(x)$ should then be easy to sketch by considering the fact that $f(x)$ is an increasing function and $g(x)$ is obtained by composing the two functions already sketched.

To sketch the graph of $y = \frac{x}{1-x^2}$ first note that there must be two vertical asymptotes. Once stationary points have been checked for it should be straightforward to complete the sketch. In this case, the asymptotes need to be considered to deduce the shape of the graph for $y = h(x)$ as the composition with $f(x)$ will lead to discontinuities. Noting again that $\tan(x + \pi) = \tan x$ the discontinuities can be resolved by translating sections of that graph vertically by a distance of π.

Question 4

(i)	 $y = f(x)$	**B1** **B1**
(ii)	$\frac{d}{dx}\left(\frac{x}{1+x^2}\right) = \frac{(1+x^2)(1)-(x)(2x)}{(1+x^2)^2}$, so stationary points when $x = \pm 1$.  $y = \frac{x}{1+x^2}$	**B1** **B1** **M1** **A1** **A1**

(iii)	 $y = \frac{x}{1-x^2}$	B1 B1 B1 B1
	 $y = h(x)$	B1 B1 B1 B1 B1